

Chiral quark model with infrared cut-off for the description of meson properties in hot matter

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Abstract. A simple chiral quark model of the Nambu–Jona-Lasinio (NJL) type with a quark confinement mechanism is constructed for the description of the light-meson sector of QCD at finite temperature. Unphysical quark production thresholds in the NJL model are excluded by an infrared cut-off in the momentum integration within quark loop diagrams. This chiral quark model satisfies the low-energy theorems. Using the vacuum masses and decay widths of π - and ρ -mesons for fixing the model parameters, the properties of the σ -meson are derived. Within the Matsubara formalism, the model is systematically extended to finite temperatures where chiral symmetry restoration due to a dropping constituent quark mass entails a vanishing of the infrared cut-off (deconfinement) at the pion Mott temperature $T_c = 186$ MeV. Besides the pion mass and weak decay constant, the masses, coupling constants and decay widths of σ - and ρ -mesons in hot matter are investigated. The quark-antiquark decay channel of the light mesons is opened for $T > T_c$ only and becomes particularly strong for the ρ -meson. The two-pion decay channel below T_c has almost constant width for the ρ -meson up to T_c , but for the σ -meson it closes below T_c such that a scalar meson state with vanishing width is obtained as a precursor of the chiral/deconfinement transition.

PACS. 11.30.Rd Chiral symmetries – 12.38.Lg Other nonperturbative calculations – 13.25.-k Hadronic decays of mesons – 11.10.Wx Finite-temperature field theory

1 Introduction

One of the most interesting phenomena predicted by QCD for a hot and dense matter is the existence of the quark-gluon plasma (QGP) phase [1], where hadrons do not exist as bound states, and the strongly interacting matter should be described in terms of QCD fundamental fields: quarks and gluons. A number of modern experiments have been already directed on the search of signals that can be interpreted as evidence for the existence of the QGP, and still new projects are upcoming. The most straightforward experiment, where QGP is hoped to be found, is a collision of heavy ions with ultrarelativistic energies, where hadrons form so hot and dense matter that the conditions necessary for QGP creation are fulfilled. However, one cannot tell certainly if QGP was seen or not without laying a proper theoretical ground for such an experiment. Actually, one needs to know what is to be expected in order to unambiguously detect whether the QGP is formed. Taking into account that a direct modelling of QGP from QCD is not available at present, one needs an appropriate quark model which would reflect the most important fea-

tures of strong interactions, on the one hand, and would be tractable, on the other hand. Among the various approaches, one can select the Nambu–Jona-Lasinio (NJL) model.

The Nambu–Jona-Lasinio model is a convenient semi-phenomenological quark model for the description of the low-energy meson physics [2–6]. Within this model the mechanism of spontaneous breaking of chiral symmetry (SBCS) is realized in a simple and transparent way, and the low-energy theorems are fulfilled.

However, the ordinary NJL model fails to prevent hadrons from decaying into free quarks, which makes the realistic description of hadron properties on their mass shell questionable. The solution to this problem seems to be a very difficult task, and different methods have been proposed for this purpose [7–12]. In our previous papers [13–15], a chiral quark model was suggested where unphysical quark-antiquark thresholds were eliminated by means of an infrared (IR) cut-off. As a result, the pole of the integrand in a quark loop integral turned out to be outside the integration interval, so that no imaginary part occurs.

This method of modeling the phenomenon of confinement is based on the idea of combining the NJL [2–6] and bag models [16–18]. Together with the ultraviolet (UV)

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As usual, one applies a bosonization procedure [3, 4] to the quark Lagrangian and obtains its equivalent representation in terms of the scalar (σ), pseudoscalar ($\vec{\pi}$), vector ($\vec{\rho}_\mu$) and axial-vector ($\vec{a}_{1\mu}$) meson fields

$$\mathcal{L}_{\text{mes}} = -\frac{\tilde{\sigma}^2 + \vec{\pi}^2}{2G_1} + \frac{\vec{\rho}_\mu^2 + \vec{a}_{1\mu}^2}{2G_2} - i\text{Tr} \ln \left\{ 1 + \frac{1}{i\hat{\not{D}} - m} [\sigma + i\gamma_5 \vec{\tau} \vec{\pi} + \vec{\tau} \hat{\not{\rho}} + \gamma_5 \vec{\tau} \hat{\not{a}}_1] \right\}. \quad (2)$$

Here, the scalar fields σ and $\tilde{\sigma}$ are connected by the relation

$$-m^0 + \tilde{\sigma} = -m + \sigma. \quad (3)$$

The vacuum expectation value of $\tilde{\sigma}$ is nonzero after SBCS and a new notation for the fluctuating part σ of the scalar field with zero vacuum expectation value, $\langle \sigma \rangle_0 = 0$, is introduced here. The vector $\hat{\not{\rho}}$ and axial-vector $\hat{\not{a}}_1$ fields are members of the Dirac algebra: $\hat{\not{\rho}} = \vec{\rho}_\mu \gamma^\mu$, $\hat{\not{a}}_1 = \vec{a}_{1\mu} \gamma^\mu$. Then, from the condition

$$\left. \frac{\delta \mathcal{L}_{\text{mes}}}{\delta \sigma} \right|_{\sigma=0, \vec{\pi}=0} = 0, \quad (4)$$

one obtains the gap equation

$$m^0 = m[1 - 8G_1 I_1^A(m)] = m + 2G_1 \langle \bar{q}q \rangle_0, \quad (5)$$

where $\langle \bar{q}q \rangle_0$ is the chiral quark condensate. The quantity $I_1^A(m)$ is obtained from the quadratically divergent integral $I_1^\infty(m)$ by regularization with the UV cut-off Λ ,

$$I_1^A(m) = -i \frac{N_c}{(2\pi)^4} \int \frac{d^4 k}{m^2 - k^2 - i\varepsilon} \theta(\Lambda^2 - |k_\perp^2|) = \frac{N_c}{(2\pi)^2} \int^\Lambda dk \frac{k^2}{E_k} = \frac{N_c}{8\pi^2} \left[\Lambda \sqrt{\Lambda^2 + m^2} - \ln \left(\frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} \right) \right], \quad (6)$$

where $E_k = \sqrt{k^2 + m^2}$, k_\perp is the 4-momentum of a quark, transverse to an arbitrary momentum P ($P^2 \neq 0$) (see [23, 24]):

$$k_{\perp\mu} = k_\mu - P_\mu \frac{P \cdot k}{P^2}, \quad (7)$$

so that for $P = (M, 0, 0, 0)$ one has $k_\perp = (0, k_1, k_2, k_3)$; N_c is the number of colors. For all applications of the model, we do not introduce an IR cut-off in $I_1^A(m)$.

Now let us consider the free part of Lagrangian (2) for pion fields in the quark one-loop approximation (see fig. 2)¹

$$\mathcal{L}_\pi^{(2)} = -\frac{\vec{\pi}^2}{2} \left\{ \frac{1}{G_1} - 8I_1^A(m) - 4P^2 I_2^{(\lambda_P, \Lambda)}(P, m) \right\}, \quad (8)$$

¹ The expression enclosed in parentheses can be written in the form $1/G_1 + \Pi_\pi(p)$, where $\Pi_\pi(p)$ is the pion polarization operator.

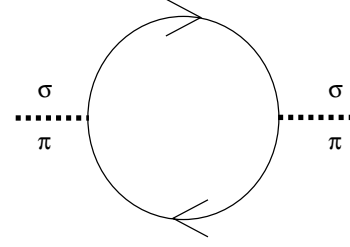


Fig. 2. The quark-loop diagram for the polarization operator of σ and π .

where $I_2^{(\lambda_P, \Lambda)}(P, m)$ is obtained from the logarithmically divergent integral $I_2^{(0, \infty)}(P, m)$ by applying IR and UV cut-offs

$$I_2^{(\lambda_P, \Lambda)}(P, m) = -i \frac{N_c}{(2\pi)^4} \int \frac{\theta(\Lambda^2 - |k_\perp^2|) \theta(|k_\perp^2| - \lambda_P^2) d^4 k}{(m^2 - k^2 - i\varepsilon)(m^2 - (k - P)^2 - i\varepsilon)} = \frac{N_c}{2\pi^2} \int_{\lambda_P}^\Lambda dk \frac{k^2}{E_k(4E_k^2 - P^2)}. \quad (9)$$

Here, P is the momentum of a bound $\bar{q}q$ state (meson) and λ_P is an infrared cut-off introduced in order to remove unphysical quark-antiquark production thresholds, see sect. 3. The last integration in (9) is done in the rest frame of a meson ($\mathbf{P} = 0$). The integral $I_2^{(\lambda_P, \Lambda)}(P, m)$ thereby depends on a Lorentz invariant, the meson mass M . Further, we prefer to consider $I_2^{(\lambda_P, \Lambda)}(P, m)$ as a function of M :

$$I_2^{(\lambda_M, \Lambda)}(M, m) \equiv I_2^{(\lambda_P, \Lambda)}(P, m) \Big|_{P=(M, 0, 0, 0)}. \quad (10)$$

To express (8) through physical fields, we renormalize the pion

$$\vec{\pi} = g_\pi(M_\pi) \vec{\pi}^r, \quad (11)$$

$$g_\pi(M_\pi) = \left[4I_2^{(\lambda_{M_\pi}, \Lambda)}(M_\pi, m) \right]^{-1/2}. \quad (12)$$

For the pion, there is also an additional renormalization factor \sqrt{Z} appearing after we take into account $\pi - a_1$ transitions [4]:

$$\bar{g}_\pi = g_\pi \sqrt{Z}, \quad Z^{-1} = 1 - \frac{6m^2}{M_{a_1}^2}, \quad (13)$$

where $M_{a_1} = 1230$ MeV is the mass of the a_1 -meson. Thus, we obtain the following expression for the pion mass:

$$M_\pi^2 = \bar{g}_\pi^2(M_\pi) \left[\frac{1}{G_1} - 8I_1^A(m) \right]. \quad (14)$$

It can be given the form of the Gell-Mann–Oakes–Renner relation

$$M_\pi^2 \approx -2 \frac{m^0 \langle \bar{q}q \rangle_0}{F_\pi^2}, \quad (15)$$

where the Goldberger-Treiman relation (see (24) below) and the gap equation (5) have been used. One can see that

this pion mass formula is in accordance with the Goldstone theorem since, for $m^0 = 0$, the pion mass vanishes and the pion becomes a Goldstone boson.

3 The σ -meson and IR confinement

The free part of Lagrangian (2) for the σ -meson in the one-loop approximation (see fig. 2) has the following form:

$$\mathcal{L}_\sigma^{(2)} = -\frac{\sigma^2}{2} \left\{ \frac{1}{G_1} - 8I_1^\Lambda(m) - 4(P^2 - 4m^2)I_2^{(\lambda_{M_\sigma}, \Lambda)}(P, m) \right\}. \quad (16)$$

After the renormalization of the σ field,

$$\sigma = g_\sigma(M_\sigma)\sigma^r, \quad (17)$$

$$g_\sigma(M_\sigma) = \left[4I_2^{(\lambda_{M_\sigma}, \Lambda)}(M_\sigma, m) \right]^{-1/2}, \quad (18)$$

one obtains the expression for the σ -meson mass

$$M_\sigma^2 = g_\sigma^2(M_\sigma) \left[\frac{1}{G_1} - 8I_1^\Lambda(m) \right] + 4m^2. \quad (19)$$

Now, let us consider more carefully the integral

$$I_2^{(\lambda_{M_\sigma}, \Lambda)}(M_\sigma, m) = \frac{N_c}{2\pi^2} \int_{\lambda_{M_\sigma}}^\Lambda dk \frac{k^2}{E_k(4E_k^2 - M_\sigma^2)}. \quad (20)$$

If $\lambda_{M_\sigma} = 0$, this integral has an imaginary part. Indeed, the integrand in (20) is singular when its denominator is equal to zero:

$$4E_k^2 - M_\sigma^2 = 0. \quad (21)$$

The imaginary part appears when the singularity ($k = \frac{1}{2}\sqrt{M_\sigma^2 - 4m^2}$) lies within the integration interval. Therefore, when one applies the IR cut-off

$$\lambda_P = [m_c \theta(m - m_c) + m \theta(m_c - m)] \times \theta(P^2 - 4m_c^2) \sqrt{\frac{P^2}{4m_c^2} - 1}, \quad (22)$$

the denominator of integral (20) has no zero if the new model parameter m_c is smaller than the constituent quark mass. Thus, integral (20) is real, and the quark-antiquark production threshold is absent. We consider this property as a criterion for the quark confinement. It is different from that of the absence of real mass poles in the quark propagator, which is employed within the DSE approach [10, 11]. The parameter m_c is unique for all mesons and provides the $\bar{q}q$ thresholds after the temperature exceeds the critical value T_c . The temperature dependence of the meson properties following from this definition of the model will be investigated in detail in sect. 6.

4 Model parameters

In the present model, there are five parameters: the constituent quark mass m , the 3D UV cut-off parameter Λ ,

the scalar (pseudoscalar) four-quark coupling constant G_1 , the vector (axial-vector) four-quark coupling constant G_2 , and the parameter m_c . To fix our parameters, we use only four observables [25]: the pion weak decay constant $F_\pi = 93$ MeV, the $\rho \rightarrow \pi\pi$ decay constant $g_\rho^{\text{exp}} = 6.14$, the pion mass $M_\pi = 140$ MeV, the ρ -meson mass $M_\rho = 770$ MeV, and the model parameter m_c , determined as $m_c = m(T_c)$. The value of T_c shall be the temperature above which the lightest meson can decay into free quarks (pion Mott temperature), which is defined by the formula

$$2m(T_c) = M_\pi(T_c). \quad (23)$$

We find that $T_c \approx 186$ MeV, and $m_c \approx 86$ MeV, see sect. 6. To fix m , Λ , G_1 , and G_2 , we use this value of m_c and the following four equations:

1) The Goldberger-Treiman relation

$$\frac{m}{F_\pi} = \bar{g}_\pi(M_\pi) = g_\pi(M_\pi)\sqrt{Z}. \quad (24)$$

2) The $\rho^0 \rightarrow \pi^+\pi^-$ decay width. The amplitude of this process is of the form

$$A_{\rho \rightarrow 2\pi} = ig_\rho^{\text{exp}}(p_{\pi^+} - p_{\pi^-})^\nu \rho_\nu^0 \pi^+ \pi^-. \quad (25)$$

In the one-loop approximation, we obtain the following expression for g_ρ^{exp} :

$$g_\rho^{\text{exp}} = 4Z^{-1}g_\rho(M_\rho)\bar{g}_\pi^2(M_\pi) \left[I_2^{(\lambda_{M_\rho}, \Lambda)}(M_\rho, m) + 2M_\pi^2 J_{3V}^{\lambda_{M_\rho}}(M_\rho, M_\pi, m) \right], \quad (26)$$

where $g_\rho(M_\rho) = \left[\frac{2}{3}I_2^{(\lambda_{M_\rho}, \Lambda)}(M_\rho, m) \right]^{-1/2}$ and the integral $J_{3V}^{\lambda_{M_\rho}}(M_\rho, M_\pi, m)$ is

$$J_{3V}^{\lambda_{M_\rho}}(M_\rho, M_\pi, m) = \frac{1}{M_\rho^2 - 4M_\pi^2} \left(I_2^{(\lambda_{M_\rho}, \Lambda)}(M_\rho, m) - \tilde{I}_2^{(\lambda_{M_\rho}, \Lambda)}(M_\pi, m|M_\rho) - M_\pi^2 I_3^{\lambda_{M_\rho}}(M_\rho, M_\pi, m) \right), \quad (27)$$

where the $\tilde{I}_2^{(\lambda_{M_\rho}, \Lambda)}(M_\pi, m|M_\rho)$ and $I_3^{\lambda_{M_\rho}}(M_\rho, M_\pi, m)$ integrals are given below:

$$\tilde{I}_2^{(\lambda_{M_\rho}, \Lambda)}(M_\pi, m|M_\rho) = \frac{N_c}{16\pi^2} \int_{\lambda_{M_\rho}}^\Lambda \frac{kdk}{E_k|\vec{p}|} \times \ln \left(\frac{(M_\pi^2 + 2k|\vec{p}|)^2 - E_k^2 M_\rho^2}{(M_\pi^2 - 2k|\vec{p}|)^2 - E_k^2 M_\rho^2} \right), \quad (28)$$

$$I_3^{\lambda_{M_\rho}}(M_\rho, M_\pi, m) = \frac{N_c}{16\pi^2} \int_{\lambda_{M_\rho}}^\infty \frac{kdk}{|\vec{p}|E_k(M_\rho^2 - 4E_k^2)} \times \left[\ln \left(\frac{(M_\pi^2 + 2k|\vec{p}|)^2 - E_k^2 M_\rho^2}{(M_\pi^2 - 2k|\vec{p}|)^2 - E_k^2 M_\rho^2} \right) + \frac{M_\rho}{2E_k} \ln \left(\frac{M_\pi^4 - (E_k M_\rho - 2k|\vec{p}|)^2}{M_\pi^4 - (E_k M_\rho + 2k|\vec{p}|)^2} \right) \right]. \quad (29)$$

Here, $|\vec{p}| = \sqrt{M_\rho^2/4 - M_\pi^2}$ is the 3-momentum of a pion after the decay of a ρ -meson in the rest frame of ρ .

The factor Z^{-1} appears due to π - a_1 transitions (see [4]). From these two equations one can find m and Λ .

- 3) The coupling constant G_1 is determined by the pion mass formula

$$M_\pi^2 = \bar{g}_\pi^2(M_\pi) \left[\frac{1}{G_1} - 8I_1^\Lambda(m) \right]. \quad (30)$$

- 4) The coupling constant G_2 is found from the mass formula for M_ρ [4]

$$M_\rho^2 = \frac{g_\rho^2(M_\rho)}{4G_2} = \frac{3}{8G_2 I_2^{(\lambda_{M_\rho}, \Lambda)}(M_\rho, m)}. \quad (31)$$

From the gap equation (5), one gets the current quark mass m^0 . The results of the parameter fixing procedure described above are summarized in table 1.

Table 1. Model parameters $m_c, m, \Lambda, G_1, G_2$, the current quark mass m^0 , and the mass and width of the σ -meson.

m_c (MeV)	m (MeV)	m^0 (MeV)	Λ (GeV)
86	242	2.1	1.09
G_1 (GeV ⁻²)	G_2 (GeV ⁻²)	M_σ (MeV)	Γ_σ (MeV)
2.98	11.8	500	205

For the experimental values of the mass and width of the σ -meson, there is a wide uncertainty. The average limits for the mass are reported to be from 400 MeV to 1200 MeV (see [25–27]), and for the width: from 600 MeV to 1000 MeV. However, smaller values were also obtained: 290 ± 54 MeV [28], 119 ± 13 MeV [29].

5 The σ -meson mass and the decay $\sigma \rightarrow 2\pi$

The mass of the σ -meson is given by eq. (19). Using this formula for $m_c = 86$ MeV (see sect. 6), we obtain

$$M_\sigma = 500 \text{ MeV}. \quad (32)$$

The decay $\sigma \rightarrow 2\pi$ is described by the quark triangle diagram (see fig. 3).

The amplitude of the process $\sigma \rightarrow 2\pi$ has the form

$$A_{\sigma \rightarrow 2\pi} = 8mg_\sigma(M_\sigma) \bar{g}_\pi^2(M_\pi) \left[I_2^{(\lambda_{M_\sigma}, \Lambda)}(M_\sigma, m) + \mathcal{J}(M_\sigma, M_\pi, m) \right] \sigma \vec{\pi}^2, \quad (33)$$

where

$$\mathcal{J}(M_\sigma, M_\pi, m) = -\frac{1}{2} (M_\sigma^2 - 2M_\pi^2) I_3^{\lambda_{M_\sigma}}(M_\sigma, M_\pi, m). \quad (34)$$

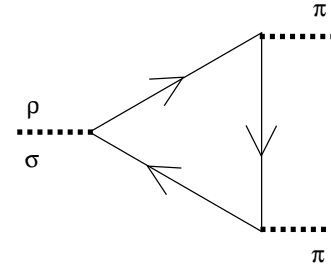


Fig. 3. The triangle quark diagram describing the decay of ρ - and σ -mesons into two pions.

Then the decay width of the σ -meson is equal to

$$\Gamma_{\sigma \rightarrow 2\pi} = \frac{3}{2\pi} \left(\frac{m^3(1+\delta)}{g_\sigma(M_\sigma) F_\pi^2 M_\sigma} \right)^2 \sqrt{M_\sigma^2 - 4M_\pi^2} = 205 \text{ MeV}, \quad (35)$$

where

$$\delta = \frac{\mathcal{J}(M_\sigma, M_\pi, m)}{I_2^{(\lambda_{M_\sigma}, \Lambda)}(M_\sigma, m)} = -0.33. \quad (36)$$

Therefore, one can see that our estimates for the σ -meson mass are in agreement with experimental data [25] (see also [26–29]) $M_\sigma^{\text{exp}} = (400\text{--}1200)$ MeV. Let us note that the corrections coming from \mathcal{J} are important for the calculation of the decay width. Indeed, a similar contribution to the $\rho \rightarrow \pi\pi$ decay width (see (27)) is small, whereas, in the case of the decay $\sigma \rightarrow \pi\pi$, it makes 30% of the amplitude and decreases the decay width by half. From this, one can conclude that the NJL model with the IR cut-off satisfies both of the low-energy theorems together with SBCS and gives a satisfactory description of the low-energy physics of the scalar, pseudoscalar and vector mesons.

6 Finite temperature case

An interesting application of our model is the description of meson properties in a hot and dense medium. The standard NJL model has been already used for this purpose in [6, 30, 31], where the temperature dependence of quark and meson masses and of Yukawa coupling constants was found.

The calculation of the constituent quark and meson masses at finite temperature can be done in the imaginary time formalism [32, 33]. In all the quark loop diagrams, now we sum over Matsubara frequencies $\omega_n = (2n+1)\pi T$ instead of integrating over the energy component of the internal quark 4-momentum. As a result, for the integral $I_1^A(m)$ one has

$$I_1^A(m, T) = -i \frac{N_c}{(2\pi)^4} \int d^4k \frac{\theta(\Lambda^2 - |k_\perp^2|)}{m(T)^2 - k^2 - i\varepsilon} \times \tanh\left(\frac{E_{k_\perp}}{2T}\right), \quad (37)$$

$$E_{k_\perp} = \sqrt{|k_\perp^2| + m^2}. \quad (38)$$

The integral $I_2^{(\lambda_{M,A})}(M, m, T)$ for a meson at rest ($P = (M, 0, 0, 0)$) in the rest frame of the heat bath is given by

$$I_2^{(\lambda_{M,A})}(M, m, T) = -i \frac{N_c}{(2\pi)^4} \int d^4k \frac{\theta(\Lambda^2 - |k_\perp^2|) \theta(|k_\perp^2| - \lambda_M^2)}{[m(T)^2 - k^2][m(T)^2 - (k+P)^2 - i\varepsilon]} \times \tanh\left(\frac{E_{k_\perp}}{2T}\right) \Big|_{P=(M,0,0,0)}. \quad (39)$$

For the integrals \tilde{I}_2 and I_3 , one obtains:

$$\tilde{I}_2^{(\lambda_{M\rho,A})}(M_\rho, m, T|M_\rho) = \frac{N_c}{16\pi^2} \int_{\lambda_{M\rho}}^{\Lambda} dk \frac{k \tanh\left(\frac{E_k}{2T}\right)}{E_k |\vec{p}|} \times \ln\left(\frac{(M_\pi^2 - 2k|\vec{p}|)^2 - E_k^2 M_\rho^2}{(M_\pi^2 + 2k|\vec{p}|)^2 - E_k^2 M_\rho^2}\right), \quad (40)$$

$$I_3^{\lambda_{M\rho}}(M_\rho, M_\pi, m, T) = \frac{N_c}{16\pi^2} \int_{\lambda_{M\rho}}^{\Lambda} dk \frac{k \tanh\left(\frac{E_k}{2T}\right)}{|\vec{p}| E_k (M_\rho^2 - 4E_k^2)} \times \left[\ln\left(\frac{(M_\pi^2 + 2k|\vec{p}|)^2 - E_k^2 M_\rho^2}{(M_\pi^2 - 2k|\vec{p}|)^2 - E_k^2 M_\rho^2}\right) + \frac{M_\rho}{2E_k} \ln\left(\frac{M_\pi^4 - (E_k M_\rho - 2k|\vec{p}|)^2}{M_\pi^4 - (E_k M_\rho + 2k|\vec{p}|)^2}\right) \right]. \quad (41)$$

The dependence of the constituent quark mass on the temperature is obtained from the gap equation (5) where integral I_1 is already T -dependent. After we know the temperature dependence of m , I_1 , I_2 , \tilde{I}_2 , and I_3 , using formulas (13), (14), (18), (19), (24), and (31), we can determine the temperature dependence of F_π , quark and meson masses, coupling constants, and meson decay widths. The results are shown in figs. 4–7.

The $\sigma \rightarrow \pi\pi$ decay width is calculated by the formula

$$\Gamma_{\sigma \rightarrow 2\pi}(T) = \frac{3|A_{\sigma \rightarrow 2\pi}(T)|^2}{32\pi M_\sigma} \sqrt{1 - \frac{4M_\pi^2}{M_\sigma^2}} \coth\left(\frac{M_\sigma}{4T}\right), \quad (42)$$

$$A_{\sigma \rightarrow 2\pi}(T) = \frac{2m(T)Z[1 + \delta] \sqrt{I_2^{(\lambda_{M_\sigma,A})}(M_\sigma, m, T)}}{I_2^{(\lambda_{M_\pi,A})}(M_\pi, m, T)}, \quad (43)$$

where δ is defined in (36). The cotangent in (42) appeared due to the interaction with the pion gas in the final state. Analogously, one has for the ρ -meson:

$$\Gamma_{\rho \rightarrow 2\pi}(T) = \frac{|A_{\rho \rightarrow 2\pi}(T)|^2 M_\rho}{48\pi} \left(1 - \frac{4M_\pi^2}{M_\rho^2}\right)^{\frac{3}{2}} \coth\left(\frac{M_\rho}{4T}\right), \quad (44)$$

$$A_{\rho \rightarrow 2\pi}(T) = \frac{\sqrt{3I_2^{(\lambda_{M_\rho,A})}(M_\rho, m, T)}}{\sqrt{2I_2^{(\lambda_{M_\pi,A})}(M_\pi, m, T)}} \times \left(1 + \frac{2M_\pi^2 J_{3V}^{\lambda_{M_\rho}}(M_\rho, M_\pi, m, T)}{I_2^{(\lambda_{M_\rho,A})}(M_\rho, m, T)}\right). \quad (45)$$

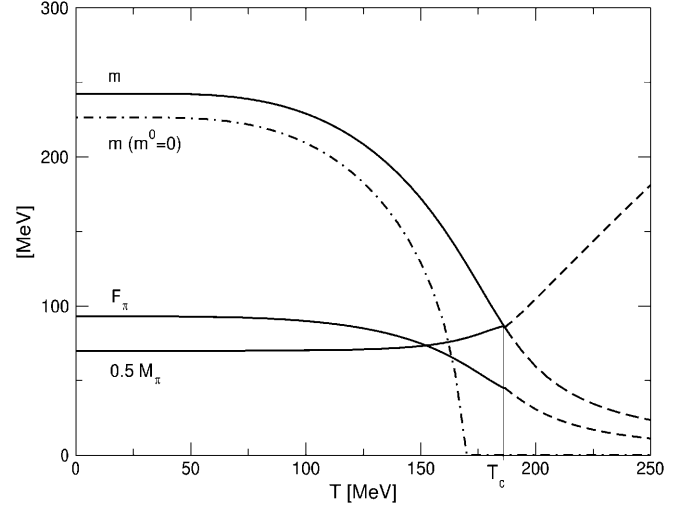


Fig. 4. The dynamical quark mass m : physical and in the chiral limit ($m^0 = 0$, dash-dotted line), the pion weak decay constant F_π , and half of the pion mass M_π .

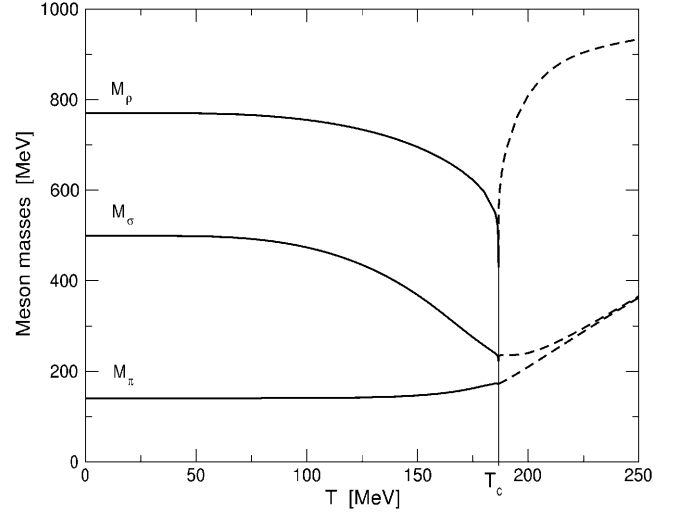


Fig. 5. Masses of σ , π , and ρ . The region above T_c is represented by dashed lines.

The quantity $J_{3V}^{\lambda_{M\rho}}(M_\rho, M_\pi, m, T)$ is derived from $J_{3V}^{\lambda_{M\rho}}(M_\rho, M_\pi, m)$ (see (27)) by replacing all integrals with ones depending on temperature.

The critical temperature T_c is determined by the condition that the pion mass is equal to the sum of the masses of its constituents (the Mott point, see eq. (23)). From this, one finds $T_c \approx 186$ MeV and $m_c \approx 86$ MeV. Thus, the lightest meson (pion) is also allowed to decay into its constituent quarks, at $T \geq T_c$. For the IR cut-off scheme considered here, the other mesons also decay into free quarks if $T \geq T_c$.²

Finally, we have the following picture. At low T , the σ - and ρ -mesons decay mostly into two pions. The pion is stable since electroweak decay channels can be neglected here

² Note that in the chiral limit ($m^0 = 0$), chiral symmetry is restored at T_c .

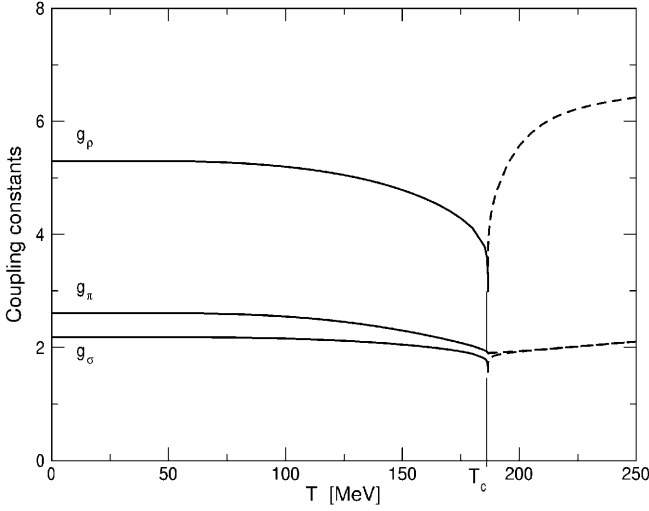


Fig. 6. The coupling constants g_π , g_σ , and g_ρ as functions of the temperature. The region above T_c is represented by dashed lines.

compared to the strong ones. At higher T , the ρ -meson still has a noticeable decay width. Unlike the ρ -meson, the width of the σ -meson first rises at $T = 100$ – 150 MeV and then falls down to zero near $T = 170$ MeV. An increasing of the width is due to the interaction with the pion gas in the final state, which leads to an additional factor increasing with temperature. Above T_c , all mesons are allowed to decay into quark-antiquark pairs, and the ρ -meson also decays into pion-like quark-antiquark correlations. It is interesting to note that the σ -meson is stable in the temperature range from 170 MeV to T_c . Here, only its electroweak decays are possible, they are small and can be neglected. Thus, we have obtained almost stable scalar meson states freely propagating through hot matter as a precursor of the chiral/deconfinement transition.

To estimate the decay widths of π -, σ - and ρ -mesons into free quarks, one should evaluate the imaginary part of the corresponding meson propagator that appears if $T > T_c$. One thereby has:

$$\Gamma_{\pi \rightarrow \bar{q}q}(T) = \frac{M_\pi \text{Im} I_2^{(\lambda_{M_\pi}, A)}(M_\pi, m, T)}{\text{Re} I_2^{(\lambda_{M_\pi}, A)}(M_\pi, m, T)}, \quad (46)$$

$$\Gamma_{\sigma \rightarrow \bar{q}q}(T) = \frac{[M_\sigma^2 - 4m(T)^2] \text{Im} I_2^{(\lambda_{M_\sigma}, A)}(M_\sigma, m, T)}{M_\sigma \text{Re} I_2^{(\lambda_{M_\sigma}, A)}(M_\sigma, m, T)}, \quad (47)$$

$$\Gamma_{\rho \rightarrow \bar{q}q}(T) = \frac{M_\rho \text{Im} I_2^{(\lambda_{M_\rho}, A)}(M_\rho, m, T)}{\text{Re} I_2^{(\lambda_{M_\rho}, A)}(M_\rho, m, T)}. \quad (48)$$

The results are shown in fig. 7 by dashed lines.

7 Discussion and conclusion

In this paper, we have investigated an extension of the NJL model for the light nonstrange meson sector of QCD, where the interaction of u - and d -quarks is represented by

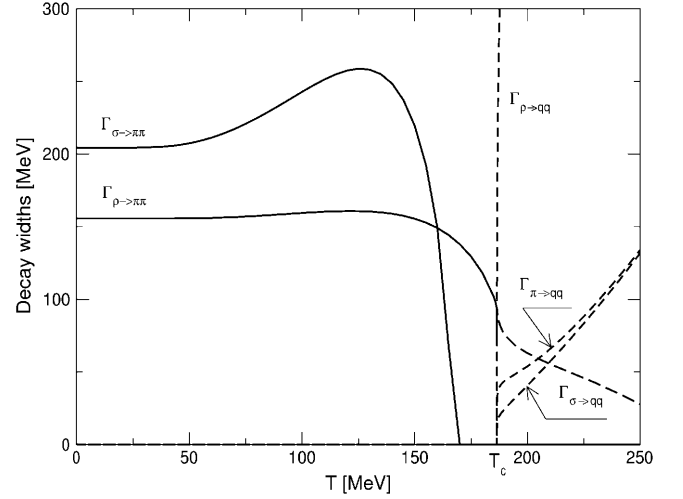


Fig. 7. The decay widths of π , σ and ρ ($\pi\pi$ and $\bar{q}q$ channels). The region above T_c is represented by dashed lines.

four-fermion vertices and the phenomenon of quark confinement is taken into account through the elimination of nonphysical quark-antiquark thresholds. This extension of the NJL model describes properties of the π -, σ - and ρ -mesons in satisfactory agreement with experiments and with low-energy theorems. The model parameters are obtained by fitting the model so that it reproduces the experimental values of the pion and ρ -meson masses, the pion decay constant F_π , and the ρ -meson decay constant g_ρ . Moreover, it has been shown that, for the π -, σ - and ρ -mesons, unphysical quark-antiquark thresholds do not appear up to the critical temperature if the IR cut-off of the form introduced here is applied.

Let us emphasize that in our model, unlike the standard NJL model [2–5], we have two cut-offs: the UV cut-off that eliminates the UV divergences and the IR cut-off which provides the confinement of quarks. The UV cut-off determines the dimension of the domain of SBCS where quarks are bosonized. It is chosen to be the same for all sorts of mesons. The second cut-off, λ_M , is introduced into the model in analogy with the bag model [16–18] and describes finite dimensions of mesons. We suppose that heavier mesons have smaller radii, therefore the IR cut-off is chosen different for various mesons, being roughly proportional to the meson mass. On the other hand, from the requirement of the absence of quark-antiquark thresholds for $T < T_c$, we determine more certainly the form of the IR cut-off. It is easy to make sure that the IR cut-off of the form (22) satisfies this condition.

The critical temperature is defined as the one at which the pion mass equals the sum of the masses of constituent quarks (the so-called Mott point). Thus, after the matter reaches T_c , the pion becomes unstable as the other mesons and they all are allowed to decay into free quarks under such conditions. This scenario is provided by the IR cut-off scheme implemented in the present work. Although, only scalar, pseudoscalar, and vector mesons have been considered, the axial-vector meson can also be treated in

the same way, and no unphysical $\bar{q}q$ thresholds will appear for it if $T < T_c$.

Let us note that the introduction of the IR cut-off has not dramatically changed the basic model parameters compared to the standard NJL model case [31] with $\lambda_P = 0$. For example, the UV cut-off increased from 1.03 GeV up to 1.09 GeV, the constituent quark mass decreased from 280 MeV down to 242 MeV, the constant G_1 also decreased from 3.48 GeV^{-2} down to 2.98 GeV^{-2} . The current quark mass has almost not changed its value 2.1 MeV. The mass and width of the σ -meson, obtained in this model, are in their experimental bounds [25,28].

The mechanism of the confinement of quarks that we introduced in our model allows us to take into account the dependence of various quantities on external momenta. As a result, we managed to estimate additional contributions to the amplitudes of $\sigma \rightarrow \pi\pi$, $\rho \rightarrow \pi\pi$ that are proportional to meson masses squared. It turned out that for the decay $\rho \rightarrow \pi\pi$ these corrections were small, and for the decay $\sigma \rightarrow \pi\pi$ they made about 30% of the amplitude and decreased the decay rate by half. Taking into account these contributions may be important if one wants to describe such quantities as form factors occurring in various processes, meson radii, scattering lengths, polarizability etc.

The σ - and ρ -mesons, considered here, can play an important role as intermediate resonances in the processes occurring in the hot hadron matter created in ultrarelativistic heavy-ion collisions. In particular, a proper consideration of these states can be useful for the explanation and prediction of signals witnessing CSR and the quark deconfinement at the transition of the hadron matter to the quark-gluon plasma phase and vice versa; for example, the low-mass dilepton enhancement observed by the CERES collaboration [34,35].

All calculations in our model are performed in the Hartree-Fock approximation which does not take into account the next to $1/N_c$ contributions. For applications to the situation in heavy-ion collisions, where a hot and dense fireball of mesons (predominantly pions) is formed, it is of interest to calculate contributions coming from intermediate pion resonances in the loop diagrams. The properties of the ρ -meson can also be modified due to the cloud of pions in hot matter [36]. An analogous situation is expected for the σ -meson [37]. In our future work we suppose to investigate the next-to-leading order corrections in the $1/N_c$ expansion.

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